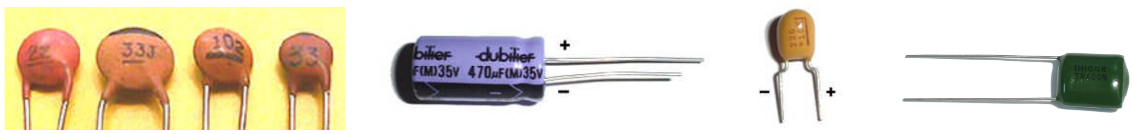
2.0 Capacitors

Capacitors show up in many circuit applications including filtering, bypassing, decoupling, and energy storage. There are several different types of capacitors that you have already encountered in your coursework including ceramic, electrolytic, tantalum and polyester as shown in Figure 2.1. Each has a range of available capacitance, tolerance, allowable frequencies, ripple current capability, voltage polarity, and voltage rating. To focus our inquiry, we will concentrate on electrolytic capacitors as they offer both large capacitance values and high ripple current capability, characteristics that we need in a buck chopper.



(a) (b) (c) (d)

Figure 2.1: Capacitor Types: (a) Ceramic; (b) Electrolytic; (c) Tantalum; (d) Polyester

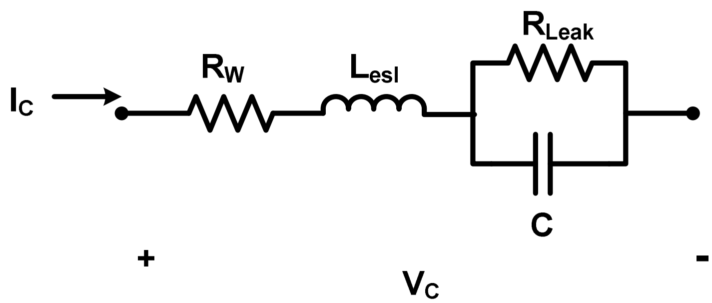
An electrolytic capacitor consists of a wound capacitor element, impregnated with liquid electrolyte (typically boric acid), connected to terminals and sealed in a can (see Figure 2.1b). The capacitor element has an etched anode foil (positive plate) coated with aluminum oxide (which acts as the dielectric), paper separators saturated with electrolyte (serving as the negative plate), and a cathode foil (allowing us to connect to the negative terminal).

A "real" capacitor is modeled by the equivalent circuit shown in Figure 2.2a. Here R_W and L_{esl} represent wire connection resistance and inductance (typically on the order of $m\Omega$ and nH), R_{leak} accounts for leakage currents through the insulator (dielectric), and C is the capacitance. Employing frequency-domain circuit reduction techniques and assuming $\omega^2 R_{leak}^2 C^2 >> 1$ (valid down to around 1Hz), we can simplify the circuit to the one pictured in Figure 2.2b. The quantity R_{esr} is termed the *Equivalent Series Resistance* (ESR) and it equals $R_{esr} = R_W + \dfrac{1}{\omega^2 R_{Leak} C^2}$. Clearly it is frequency dependent and manufacturers will typically list the ESR value at some convenient frequency like 120Hz (a standard full-wave rectifier frequency). The ESR accounts for power loss and thus heat dissipation in the capacitor. The real power loss is calculated by

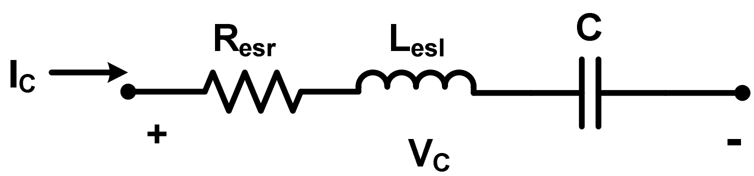
$$P_{Cap} = I_{C,rms}^2 R_{esr} \tag{2.1}$$

Thus a capacitor will have a limit to how much RMS current (and thus ripple current) that it can accommodate. The model shown in Figure 2.2b also conveys another interesting fact: at high enough frequencies, the capacitor will actually start looking like an inductor! The impedance looking into the capacitor model is given by

$$Z_{Cap} = R_{esr} + j\left(\omega L_{esl} - \frac{1}{\omega C}\right) \tag{2.2}$$



(a)



(b)

Figure 2.2: Equivalent Circuit Representations of an Electrolytic Capacitor

The frequency at which the capacitor transitions from looking capacitive to inductive is called the *self resonant frequency* and it is found as

$$\omega_{C,res} = \frac{1}{\sqrt{L_{esl}C}} \tag{2.3}$$

If we plot the impedance magnitude versus frequency for a $100\mu F$ capacitor with typical parasitic values $R_W = 20m\Omega$ and $L_{esl} = 10nH$, we get the result illustrated in Figure 2.3. The self-resonant frequency occurs at about 159kHz, implying that above that the device no longer operates like a capacitor. This fact also has interesting ramifications in terms of noise. It is common to add small capacitors in parallel with the main buck capacitor to intentionally improve the very-high frequency performance of the circuit.

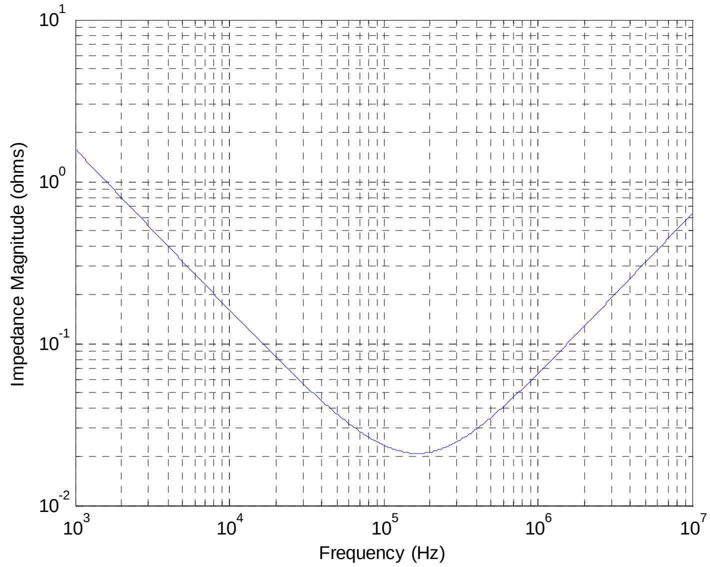


Figure 2.3: Impedance Magnitude versus Frequency for a Real Capacitor

The *dissipation factor* (DF) is often used to indicate the quality of a capacitor: it is the ratio of the capacitor's resistance to the reactance and it is principally a function of the material property of the insulator. This ratio is also referred to as the *loss tangent* and designated by $\tan\delta$ (you can see both in data sheets). For frequencies far below the self-resonant frequency where the capacitive reactance dominates, we have

$$DF = \tan\delta = \frac{R_{esr}}{\frac{1}{\omega C}} = \omega R_{esr} C \tag{2.4}$$

Thus to estimate the ESR, we rearrange and solve

$$R_{esr} \approx \frac{DF}{\omega C} \approx \frac{\tan\delta}{\omega C} \tag{2.5}$$

And so ESR varies inversely with both C and the frequency.

Thus to get a low-ESR value, one needs to either operate at high frequencies (but not beyond the self-resonant frequency) or have a large capacitance (which will tend to lower the self-resonant frequency). As one will find, there are numerous types of aluminum electrolytic capacitors sold by manufacturers, typically listed in terms of a Series. The Series will have ranges of capacitor values, rated voltages, and will normally be targeted for certain applications (you can investigate these in any vendor catalog). Let's consider some typical data from a Digikey catalog for a set of FC-series Panasonic radial-leaded aluminum electrolytic capacitors.

Table 2.1 conveys several interesting trends. First, increasing the capacitor voltage rating (and the physical size of the device) will increase the ripple current capability. Second, for a given voltage rating, increasing the capacitance does not proportionately increase the ripple current capability. For instance at 100V, five $100\mu F$ capacitors will provide 3300mA of ripple current capacity while a single $470\mu F$ capacity provides only 1920mA (not 4.7 times as much). Let's consider this comparison further, the ESR for the single $470\mu F$ capacitor is approximately $\frac{0.07}{\omega \times 470\mu F}$. The ESR for each of the five capacitors in parallel is $\frac{0.07}{\omega \times 100\mu F}$ (which is about 5 times bigger). Since these capacitors are in parallel, the equivalent resistance is 1/5 the size, so there is little net difference in ESR between the two options. How about the self-resonant frequency? Let's assume that each part has an equivalent series inductance of 20nH. Thus the $470\mu F$ capacitor will have a self-resonant frequency of 52kHz. By being in parallel, the effective capacitance of the five $100\mu F$ capacitors is $500\mu F$ while the effective inductance is $\frac{20nH}{5} = 4nH$ and therefore the self-resonant frequency becomes 113kHz. Thus, paralleling capacitors is an effective way of increasing both ripple capability and the self-resonant frequency. This is illustrated in Figure 2.4.

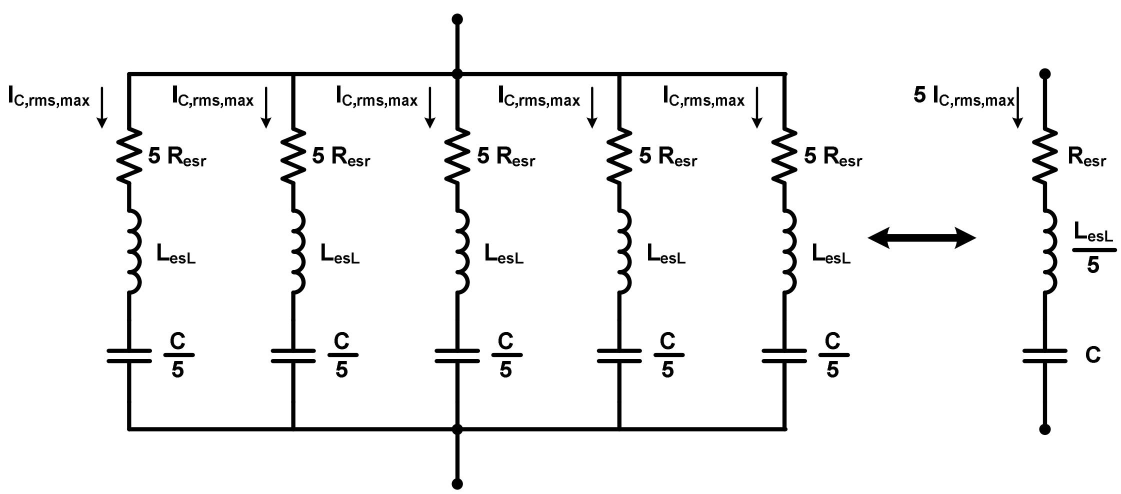


Figure 2.4: Illustration of How Paralleling Capacitors Decreases the Equivalent Series Inductance

What about the effect of the ESR on the output ripple of the buck chopper? With the majority of the inductor ripple current going down through the capacitor and thus through the ESR, we get an additional output voltage ripple component of

$$\Delta V_{out,pp,esr} = \Delta I_L R_{esr} = \frac{\Delta I_L}{I_{L,ave.\max}} I_{L,ave,\max} R_{esr} = R_{esr} r I_{L,ave,\max} \tag{2.6}$$

If we substitute our expression for the equivalent series resistance (2.5), this component of ripple becomes

$$\Delta V_{out,pp,esr} = \frac{DF}{\omega C} r I_{L,ave,\max} \tag{2.7}$$

If we compare this with equation (1.27) which gives us the peak-to-peak ripple due to the filtering influence of the capacitance, expressed here as

$$\Delta V_{out,pp,C} = \frac{\Delta I_L}{8 f_{sw} C} = \frac{\left(\frac{\Delta I_L}{I_{L,ave,\max}}\right) I_{L,ave,\max}}{8 \times \frac{2\pi f_{sw}}{2\pi} \times C} = \frac{\pi / 4}{\omega C} \times r \times I_{L,ave,\max} \tag{2.8}$$

We notice that since $\pi / 4 = 0.785$ is much larger that the dissipation factor for most electrolytic capacitors (DF < 0.2), the output ripple is dominated by the capacitor value selection. Also, since the capacitor current and voltage are 90° out of phase, you cannot simply add the effects of (2.7) and (2.8) but must combine them via

$$\Delta V_{out,pp} = \sqrt{\left(\Delta V_{out,pp,esr}\right)^2 + \left(\Delta V_{out,pp,C}\right)^2} \tag{2.9}$$

Table 1: Data for FC-Series Radial Aluminum Electrolytic Capacitors

Rated Voltage	**16V**	**35V**	**63V**	**100V**
DF	0.16	0.12	0.08	0.07
Ripple Current 100μF	290mA	555mA	535mA	671mA
Ripple Current 220μF	555mA	755mA	1050mA	1170mA
Ripple Current 470μF	730mA	1220mA	1765mA	1920mA

Thus for the buck chopper circuit, the output voltage ripple is dominated by the choice of capacitance and not the capacitor ESR.

One last final point before we leave capacitors and dig into inductors, capacitors are sold in various package types. This is important because you both want to minimize inductance and be able to connect the modules into the rest of your circuit. Some commonly available packages are shown in Figure 2.5.

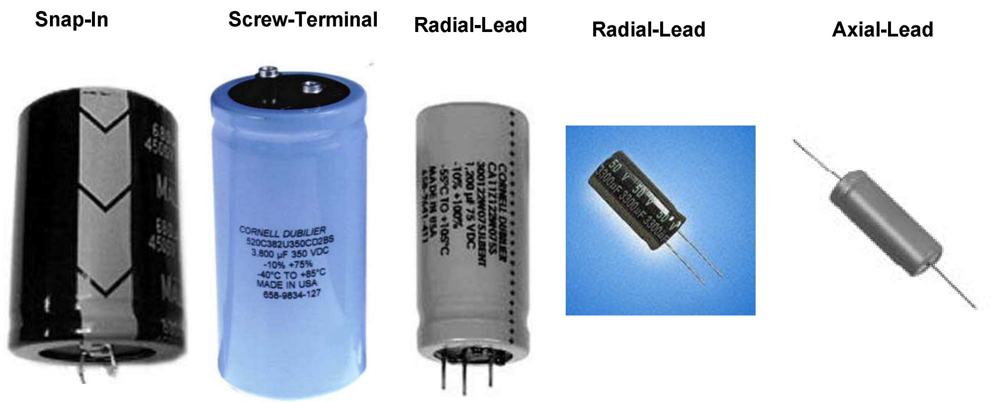


Figure 2.5: Package Options for Electrolytic Capacitors

<u>Example 2.1</u>: Consider design Example 1.5 where we wanted a buck chopper to step 24V down to 12V with a maximum output power of 100W, a switching frequency of 40kHz, $P_{crit} = 10W$ and $\Delta V_{Cpp} \leq 120mV$. We identified that $\Delta I_L = 1.67A$ and $C_{\min} = 61\mu F$. Consider that we have the following 35V Series FC electrolytic capacitors to choose from and further, assume that the equivalent series inductance of each is 20nH. Determine the best solution for the converter.

Capacitance	Ripple Current
12uF	120mA
22uF	175mA
39uF	235mA
68uF	290mA
100uF	555mA

First, let's start with the required ripple current. If we go back to (1.36), we evaluate

$$I_{C,rms} = \frac{\Delta I_L}{\sqrt{12}} = \frac{1.67A}{\sqrt{12}} = 0.482A$$

Also, since we are switching at 40kHz, we desire $f_{C,res} = \frac{\omega_{C,res}}{2\pi} = \frac{1}{2\pi\sqrt{L_{esl}C}}$ to be much less than this. OK, since we know that we need at least 61uF, we can consider the minimum number of each value of capacitance, determine the resonant frequency, and the total ripple current capability

Table 2: Computation for Example 2.1

C Value	#	C_{Tot}	$L_{esL,tot}$	$f_{C,res}$	$I_{C,rms}$
12uF	5	5 x 12uF = 60uF	20nH/5 = 4nH	325kHz	5 x 120mA = 600mA
22uF	3	3 x 22uF = 66uF	20nH/3 = 6.7nH	240kHz	3 x 175mA = 525mA
39uF	2	2 x 39uF = 78uF	20nH/2 = 10nH	180kHz	2 x 235mA = 470mA
68uF	1	1 x 68uF = 68uF	20nH/1 = 20nH	136kHz	1 x 290mA = 290mA
100uF	1	1 x 100uF = 100uF	20nH/1 = 20nH	113kHz	1 x 555mA = 555mA

What we notice from Table 2 is (1) rows 3 and 4 must be excluded since they do not have an adequate total ripple current (the values are < 482mA) and (2) the resonant frequency is highest for the parallel combination of the smallest capacitors, though the remaining three options all have resonant frequencies comfortably above 40kHz. Your decision at this point might be guided by space and/or cost: can you afford to place five 12uF capacitors on your pc-board or would you prefer fewer?

Now that we understand that the capacitor selection is NOT simply choosing a value of C (that it involves consideration of both the ripple current capability and the resultant resonant frequency), we must soldier on to investigate the selection/design of a *real inductor*!